



Faculty of Engineering and Information Technology

Topological Shape Optimization of Microstructures of Materials using Level Set Methods

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Topological shape optimization of microstructures of materials using level set methods

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Certificate of Original Authorship

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

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Publications and Conference Contributions

The following publications are part of the thesis

Peer reviewed international scientific journal publications

- [1] **Y. Wang**, Z. Luo, N. Zhang, Topological optimization of structures using a multilevel nodal density-based approximant, *CMES: Computer Modeling in Engineering & Sciences*, Vol. 84, Issue.3, pp. 229-252, 2012.
- [2] **Y. Wang**, Z. Luo, J. Wu, N. Zhang, Topology optimization for micro compliant mechanisms using the element-free Galerkin method, *Advances in Engineering Software*, Vol. 85, pp. 61-72, 2015.
- [3] **Y. Wang**, Z. Luo, N. Zhang, QF. Qing, Topological shape optimization of multifunctional tissue engineering scaffolds with level set method. *Structural and Multidisciplinary Optimization*. (Major revision, Manuscript ID SMO-14-039)
- [4] **Y. Wang**, Z. Luo, N. Zhang, Topological design for mechanical metamaterials using a multiphase level set method, *Structural and Multidisciplinary Optimization*. (Major revision, Manuscript ID SMO-15-0079)
- [5] **Y. Wang**, Z. Luo, N. Zhang, Design of micro-structured metamaterials using a multiphase level set method, *Engineering Optimization* (Major revision, Manuscript ID GENO-2014-0193)
- [6] Z. Luo, N. Zhang, **Y. Wang**, A physically meaningful level set method for topology optimization of structures, *CMES: Computer Modeling in Engineering & Sciences*, Vol. 83, Issue.1, pp. 73-96, 2012
- [7] Z. Luo, N. Zhang, **Y. Wang**, W. Gao, Topology optimization of structures using meshless density variable approximants, *International Journal for Numerical Methods in Engineering*, Vol. 93, Issue.4, pp. 443-464, 2013

Peer reviewed international scientific conference publications

- [1] **Yu Wang**, Zhen Luo. Shape and topology optimization using a nodal density-based level set method, *The 14th Asia Pacific Vibration Conference APVC 14*, 5-8 December 2011, The Hong Kong Polytechnic University.
- [2] **Y. Wang**, Z. Luo. A radial point-based meshfree Galerkin method for topology optimization, *The 4th International Conference on Computational Methods ICCM 2012*, 25-27 November, Gold Coast, Australia.
- [3] N. Zhang, **Y. Wang**, Z. Luo. Structural topology optimization using meshfree methods, *The 23rd International Congress of Theoretical and Applied Mechanics ICTAM 2012*, 19-24 August, 2012, Beijing, China.
- [4] Z. Luo, **Y. Wang**, N. Zhang, Topology optimization of lightweight structures using a meshless shepard function approximant, *The 7th Australasian Congress on Applied Mechanics ACAM 7*, 9-12 December, 2012, Adelaide, Australia.
- [5] **Y. Wang**, Z. Luo, N. Zhang, Topology optimization of compliant mechanisms using element-free methods, *The 2013 World Congress on Global Optimization WCGO 2013*, 7-12 July, 2013, The Yellow Mountains, China. *Advances in Global Optimization* Springer Proceedings in Mathematics & Statistics, 95, 2015, pp 217-226
- [6] **Y. Wang**, Z. Luo, N. Zhang, Design optimization of structures using a nodal density-based SIMP method, *The 13th Asia-Pacific Congress for Computational Mechanics, APCOM 2013*, 11-14 December, 2013, Singapore.
- [7] **Y. Wang**, Z. Luo, N. Zhang, The topological design of metamaterials using a level set method, *The 11th World Congress on Computational Mechanics WCCM XI 2014*, 21-25 July, 2014, Barcelona.

Abstract

Topology optimization has been regarded as a most promising approach in the conceptual stage of structural design. It has experienced rapid development over the past two decades and has been applied to a wide range of engineering problems. This thesis will focus on the level-set based topology optimization method, which was originally developed by Osher and Sethian in 1988 and have been successfully incorporated into the structural optimization. With the implicit representation scheme, the level set methods can be easily applied to handle the complex shape and topology changes of the structural design.

This work is divided into three parts. The first part is the necessary background for understanding the main focuses of the thesis. It includes the first four chapters: Chapter 1 provides the background the topology optimization, and overview of the current topology optimization methods, as well as application in the material design fields. Chapter 2 gives a description of the numerical homogenization method. Chapter 3 introduces the theory of the conventional level-set methods, and Chapter 4 provides details of for a parameterized level set method with numerical examples.

The second part of this thesis is about the design of mechanical/elastic metamaterials, which is contented in Chapter 5. In this part, we integrate the parameterized level set method with the numerical homogenization method for the design problems of metamaterials. Meanwhile, a multiphase level-set based scheme for designing metamaterials is proposed. In the parameterized level set method, a set of compactly supported radial basis functions (CSRBF) is employed to interpolate each implicit level set function, which transfer the most

difficult topology optimization problem into an easiest “size” optimization problem in the area of structural optimization. This method will be free of the Courant–Friedrichs–Lewy (CFL) condition and the re-initialization scheme. The propagation of the level set function can be driven by other well-developed optimization that involves gradient information. Moreover, this method can freely create new holes inside the material regions of the two-dimensional (2D) design domain. The optimal designs for mechanical metamaterials with extreme and prescribed properties are presented in this chapter as well (e.g. negative thermal expansion and negative Poisson’s ratios).

In the third part of this thesis, Chapter 6, we applied the parametric level set method and the numerical homogenization method for designing three-dimensional (3D) scaffolds for the tissue engineering. Numerical examples are used to demonstrate the effectiveness of the optimization method in designing the scaffold with a range of multifunctional properties. The efficiency, convergence and accuracy of the present methods are also highlighted.

Finally conclusions are given in Chapter 7.

Keywords: Topological shape optimization; Let set method; Homogenization method; Microstructures, Mechanical metamaterial; Tissue engineering scaffold.

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Nomenclature

Global abbreviations used in this thesis

CAMD	=	Continuous approximation of material distribution
CFL	=	Courant–Friedrichs–Lewy
CS-RBF	=	Compactly supported radial basis function
EFG	=	Element-free Galerkin
ESO	=	Evolutionary structural optimization
FEM	=	Finite element method
GCMMA	=	globally convergent method of moving asymptotes
H-J PDE	=	Hamilton–Jacobi partial differential equation
KKT	=	Karush-Kuhn-Tucker
LSFs	=	Level set functions
LSM	=	Level set-based method
MLPG	=	Meshless local Petrov-Galerkin
MLS	=	Moving least squares
MMA	=	Method of moving asymptotes
MPLSM	=	Multi-phase level set method
NPR	=	Negative Poisson’s ratios
NTE	=	Negative thermal expansion

OC	=	Optimality criteria
PDI	=	Pointwise density interpolation
PLSM	=	Parametric level set method
PIM	=	Point interpolation method
PUM	=	Partition of unity method
PDE	=	Partial differential equation
RBFs	=	Radius basis functions
RPIM	=	Radial point interpolation method
RKPM	=	Reproducing kernel particle method
SIMP	=	Solid isotropic material with penalization
SPH	=	Smooth particle hydrodynamic method
ZTE	=	Zero thermal expansion
2D	=	Two dimensional
3D	=	Three dimensional